Exercise VIII

- 1. Prove that there is a real number L such that $L^3 = 5$.
- 2. Use the Intermediate Value Theorem to prove that a plate of mashed potato can be evenly divided by a single straight vertical knife cut.



- 3. Give an example of a quadratic $x^2 + bx + c$ which has no real roots.
- 4. Give an example of a quartic $x^4 + bx^3 + cx^2 + dx + e$ which has no real roots.
- 5. Use the Intermediate Value Theorem to prove that

$$x^5 - 2x^4 + 7x^3 + 0.5x^2 - 0.7 = 0$$

has at lease one real solution.

- 6. How do we know that $x^3 15x^2 + 27x + 1$ has a maximum and a minimum value over [-2,4]? Find the maximum and minimum value respectively and explain how you know that your answers are correct.
- 7. How do we know that $\frac{17}{12}x^3 + 4x^2 x + 5$ has a maximum and a minimum value over over (i) [-3,3], (ii) [-1,1] and (iii) [2,3] respectively? Find the maximum and minimum in each case and justify your answers.