## Exercise VIII

1. Prove that there is a real number $L$ such that $L^{3}=5$.
2. Use the Intermediate Value Theorem to prove that a plate of mashed potato can be evenly divided by a single straight vertical knife cut.

3. Give an example of a quadratic $x^{2}+b x+c$ which has no real roots.
4. Give an example of a quartic $x^{4}+b x^{3}+c x^{2}+d x+e$ which has no real roots.
5. Use the Intermediate Value Theorem to prove that

$$
x^{5}-2 x^{4}+7 x^{3}+0.5 x^{2}-0.7=0
$$

has at lease one real solution.
6. How do we know that $x^{3}-15 x^{2}+27 x+1$ has a maximum and a minimum value over $[-2,4]$ ?
Find the maximum and minimum value respectively and explain how you know that your answers are correct.
7. How do we know that $\frac{17}{12} x^{3}+4 x^{2}-x+5$ has a maximum and a minimum value over over (i) $[-3,3]$, (ii) $[-1,1]$ and (iii) $[2,3]$ respectively?
Find the maximum and minimum in each case and justify your answers.

